*Journal of Engineering Physics and Thermophysics, Vol. 81, No. 5, 2008*

# CHARACTERISTIC FEATURES OF THE OSCILLATION OF SHELLS OF BODIES OF REVOLUTION IMMERSED IN A VISCOUS FLUID FLOW

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UDC 532.533

*The influence of the physicomechanical properties of large-scale shells on their oscillation in a fluid flow is investigated. It is shown that shells of length-variable rigidity may in principle ensure the phase velocity of a traveling wave along the meridian such as to transfer energy from the fluid to an elastic coating over the starting length and thus to preserve it in the form of the energy of elastic oscillations and to recover the fluid pulse on the back side.*

**Introduction.** As is known [1], a traveling wave on a shell surface can be used as a means of controlling a fluid flow. Hydrodynamic calculations for an artificially produced traveling wave [2–8] demonstrates the potential of a substantial decrease in the body resistance; however, the conditions under which a traveling wave may be sustained in the shell of finite length have not been investigated up to now. To attain the effect of the reduction in hydrodynamic resistance it is necessary to control the parameters (phase velocity and amplitude) of the traveling wave generated as a result of the interaction of an elastic shell with a fluid flow. These parameters are determined, on the one hand, by the fluid flow and, on the other, by the properties of the elastic structure. The laws that govern the traveling wave generated at the rotating nose are being investigated in the present work.

**Statement of the Boundary-Value Problem and the Method of Its Solution.** To study the behavior of a shell immersed in a fluid flow it is necessary to solve conjugate problems — of hydrodynamics and elastic deformation — as a coupled problem of hydroelasticity that includes the equations of motion of an elastic shell:

$$M\ddot{\delta} + R\ddot{\delta} + K\delta = F(t) \tag{1}$$

and the Navier–Stokes equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\Delta\mathbf{u}) = -\nabla p + \frac{1}{\mathrm{Re}}\Delta\mathbf{u}\,, \quad \mathrm{div}\,\mathbf{u} = 0\,. \tag{2}$$

The flow and the shell move simultaneously so that the normal component of the flow velocity on the shell surface is equal to the normal component of the velocity of the shell itself. Then the motion of the fluid and of the shell is governed by a system of differential equations (1), (2) with standard initial and boundary-value conditions and an additional boundary condition on the shell surface:

$$u_{\mathrm{n}}(z, r_0) = \frac{\partial w}{\partial t}\,.$$

Here the shell and fluid flow are related to a cylindrical coordinate system ($z$, $r$). The displacement of the shell along the normal $w$ is governed by the equation of motion (1), the right-hand side of which contains the pressure $p$ calculated from Eq. (2).

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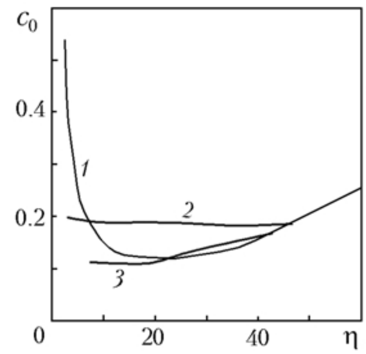


Fig. 1. Dispersion curves of a thin shell: 1) analytical solution for a shell of infinite length; 2 and 3) numerical solution for a shell of length 1 m at $E_s = E_\theta$ and $E_s = 10E_\theta$. η, m$^{-1}$.

The procedure of solving the conjugate difference problem was based on the joint use of a package of programs "Composite-2005," and the program of integration of the Navier–Stokes equations [2].

**Results of Calculations.** It is worthwhile that an analysis of the qualitative picture of axisymmetric oscillations of a shell of finite length whose edge is subjected to the action of the force changing in time by the harmonic law could be carried out in comparison with the solution of the problem of elastic wave propagation in a shell of infinite length considered in [9], where with some simplifying assumptions a dispersion equation (the dependence of the elastic wave phase velocity $c_{ph}$ on the wave number η) was obtained:

$$\left(1-\frac{1-\nu}{3}c_0^2\right)\left(1+\frac{k^2\eta^4}{12}-\frac{1-\nu}{3}c_0^2\eta^2\right)-\nu^2=0\,, \tag{3}$$

where $c_0 = c_{ph}\sqrt{\frac{3\rho}{2G}}$. The dispersion curves obtained in solving the problem (1), (2) and Eq. (3) are presented in Fig. 1. The solution of (1), (2) has demonstrated a qualitative difference between the oscillations of a finite-length shell and the propagation of waves in an infinitely long shell. First of all, the traveling wave effect does not take place at a zero damping coefficient that was adopted in [9], and the wave is reflected from the rear edge in the entire range of wave numbers. In the case of a shell of length 1 m this coefficient must not be smaller than 0.2. Calculation of the oscillations of a thin shell ($k = 0.01$) yields a dimensionless phase velocity that agrees with the solution of Eq. (3) only in order of magnitude, but there is a substantial difference in the dispersion curve. It is seen from Fig. 1 that at the elasticity moduli $E_s = E_\theta$ the phase velocity is practically constant, whereas the dispersion curves for an orthotropic shell depend on the ratio of the elasticity moduli $E_s$ and $E_\theta$, and when $E_s$ increases 10 times, the dependence of the phase velocity on the wave number becomes more noticeable. At a nonzero damping coefficient the reflection of a wave from the rear edge was also registered, but already for high wave numbers. Thus, at a damping coefficient of 0.2 and wave number equal to 50 and above a traveling wave decomposes because of its reflections, and the oscillations display a rather complex picture distorted by the presence of standing waves; here oscillations with wave number exceeding 50 do not have the shape of a traveling wave (Fig. 2). Short-wave oscillations damp less along the shell axis and are more subjected to reflection from the rear edge, which requires that the construction be specially equipped with a damper on the rear edge that must effectively damp short waves. The role of a damper can also be played by the fluid provided that the phase velocity of a traveling wave astern is higher than that of the flow, precisely what was assumed according to [8].

To study the influence of the damping coefficient on the traveling wave parameters, in particular, on its phase velocity and damping length, we considered an isotropic cylindrical shell with a unit radius and the length equal to $z_k = 1$. As the damping length we adopted the length over which the traveling wave amplitude decreases 100 times. The dimensionless damping coefficient varied from 0.15 to 0.6.

Figure 3 demonstrates the graphs of the dependences of the phase velocity $c_{ph}$ and traveling wave damping length $L$ on the damping coefficient $r$. In this case, the wave number is equal to 10 m$^{-1}$, which corresponds to the

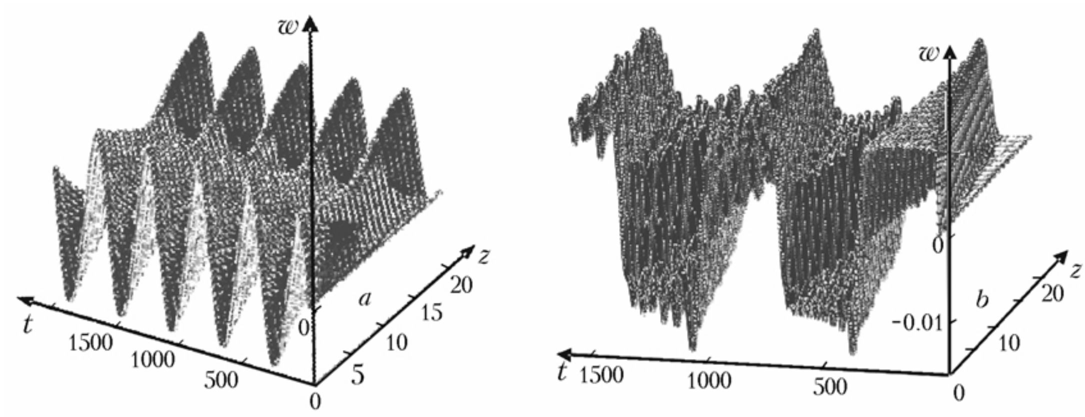


Fig. 2. Normal displacements of the shell vs. the time $t$ and axial coordinate $z$: a) traveling wave; b) decomposition of a traveling wave after reflection at the edge. $w$, $z$, m; $t$, sec.

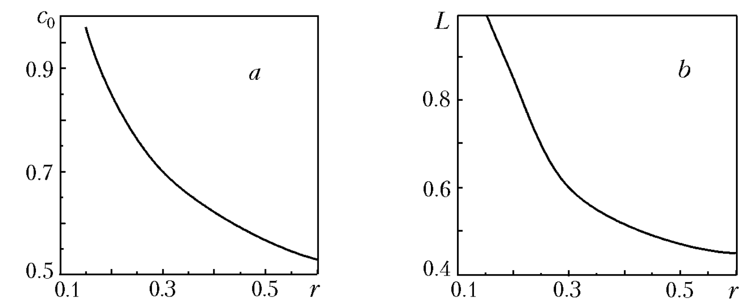


Fig. 3. Graphs of the dependence of the phase velocity $c_0$ (a) and traveling wave attenuation length $L$ (b) on the damping coefficient $r$. $L$, m; $r$, sec; $c_0$, m/sec.

phase velocity minimum on the dispersion curve. The calculations have shown that on increase in the damping coefficient both the phase velocity and the traveling wave damping length decrease, with the wave not reaching the shell edge. Here the traveling wave is excited over the entire shell. The reverse holds when the damping coefficient decreases strongly (it is less than 0.15): the phase velocity increases and the traveling wave length exceeds the shell length, leading to the reflection of this wave from the edge and to the appearance of standing waves. Therefore, variation of the damping coefficient from 0.2 to 0.3 is optimal. Such values of the coefficient can be obtained, for example, in a three-layer shell with a foam plastic filler.

We also investigated the influence of the local change in the flexural rigidity of the shell. The calculations have shown that a twofold decrease of it or less does not lead to an increase in the phase velocity in the undamaged zone. The wave velocity in the zone of attenuation decreases in this case proportionally to the decrease in the rigidity. With further decrease in the rigidity (from 2 to 4 times) the phase velocity continues to decrease, and the velocity in the undamaged zone starts to increase. Finally, in the case where the rigidity decreases 10 times and more, the phase velocities in both zones are established at constant levels: in the undamaged zone they are 2 times higher and in the damaged zone 4–5 times lower than in the entirely undamaged shell.

**Conclusions.** It is shown that shells with a rigidity variable over length can in principle provide the phase velocity of a traveling wave along the meridian such that over the starting length the energy from the fluid could be transferred to an elastic coating and thus to preserve it in the form of the energy of elastic oscillations and to recover the pulse in the fluid in the afterbody region.

To realize a traveling wave in a shell of finite length methods are needed that would ensure scattering of the energy of the wave when it passes from the frontal edge to the rear one. For this purpose, in the construction of shells one can use special damping layers, e.g., a honeycomb or foam plastic light filler.

The characteristic feature of wave processes in frame structures is the reflection of waves from the shell edges leading to the breaking of a traveling wave and formation of a complex picture that owes its origin to standing waves; this should be taken into account in selecting the physicomechanical properties of coating.

On a small local decrease in the flexural rigidity of the shell the wave velocity in the zone of attenuator decreases proportionally to the decrease in the rigidity and does not alter the velocity beyond its limits.

This work was carried out with support from the Russian Foundation for Basic Research, grant No. 06-01-00004a.

## NOTATION

$c_{\text{ph}}$, phase velocity of an elastic wave; $c_0$, dimensionless phase velocity; $E_{\text{s}}$, $E_\theta$, elasticity moduli; $F$, perturbing force; $G$, shear modulus; $K$, shell rigidity matrix; $k$, ratio of the shell thickness to its radius; $L$, attenuation length of a traveling wave; $M$, matrix of shell masses; $p$, pressure; $R$, shell damping matrix; $r$, damping coefficient; Re, Reynolds number; $r_0$, shell radius; $t$, time; $\mathbf{u}$, flow velocity vector; $u_{\text{n}}$, normal component of the vector; w, normal velocity component of the displacements of the shell; $z$, coordinate; $z_k$, shell length; $\delta$, displacement of the shell; $\eta$, wave number; $\nu$, Poisson coefficient; $\rho$, density of material. Subscripts: n, normal; ph, phase; $s$, length of the meridian arc; $\theta$, central angle in the cross section of the shell.

## REFERENCES


1. V. I. Merkulov, *Management of Liquid Motion* [in Russian], Nauka, Novosibirsk (1981).
2. S. M. Aul'chenko, Modeling of the mechanism of decreasing the hydrodynamic drag of bodies by the surface running-wave method, *Inzh.-Fiz. Zh.*, **76**, No. 6, 24–28 (2003).
3. S. M. Aulchenko and Yu. Nikulichev, Modeling of the mechanism of hydrodynamic drag reduction by the method of a surface running wave, in: *Proc. Int. Conf. on the Methods of Aerophysics Research*, 28 June–2 July, 2004, Novosibirsk (2004), Pt. 4, pp. 44–49.
4. S. M. Aul'chenko, Modeling of reduction in the resistance of a body of revolution in a viscous fluid, *Inzh.-Fiz. Zh.*, **78**, No. 3, 193–195 (2005).
5. S. M. Aul'chenko, Control of flow past a body of revolution, minimizing the body's drag in a viscous fluid, *Inzh.-Fiz. Zh.*, **79**, No. 5, 109–111 (2006).
6. S. M. Aul'chenko, V. O. Kaledin, E. V. Reshetnikova, and Yu. V. Anikina, Modeling of the mechanism underlying the reduction of the resistance of shells immersed in a viscous liquid flow, in: *Boundary-Value Problems and Mathematical Modeling*, *Proc. 9th All-Union Scientific Conf.*, Novokuznetsk, NFI Kem GU (2006), pp. 8–14.
7. S. M. Aulchenko, V. O. Kaledin, and Yu. Nikulichev, Modeling of the mechanism of drag reduction of the bodies in the viscous fluid, in: *Proc. Int. Conf. on the Methods of Aerophysics Research*, 5–10 February, 2007, Novosibirsk (2007), Pt. 1, pp. 13–16.
8. S. M. Aul'chenko, V. O. Kaledin, and Yu. V. Anikina, Modeling of the mechanism underlying the reduction in the resistance of the shells of the bodies of revolution immersed in a viscous liquid flow, *Pis'ma Zh. Tekh. Fiz.*, **33**, Issue 17, 83–88 (2007).
9. A. G. Gorshkov and V. I. Pozhuev, *Stationary Problems of the Dynamics of Multilayer Structures* [in Russian], Mashinostroenie, Moscow (1992).